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IMPROVED METHODS FOR LARGE SCALE STRUCTURAL SYNTHESIS.(U)  
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FINAL REPORT

IMPROVED METHODS FOR LARGE SCALE STRUCTURAL SYNTHESIS

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Further research is required to refine these methods and substantiate the initial successes.

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by

M. Pappas

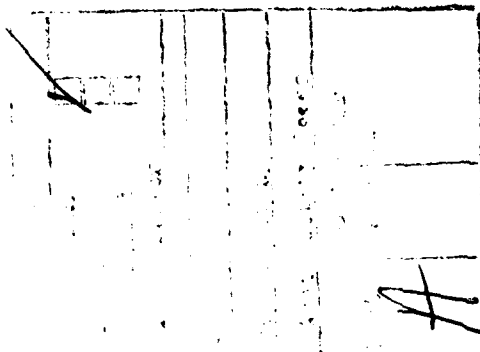
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## I. INTRODUCTION

The structural optimization problem may be posed by the following:<sup>1</sup>

Find

$$\min f(x_i) \quad i = 1, 2, \dots, I \quad (1)$$

subject to the conditions

$$g(x_i) \leq 0 \quad j = 1, 2, \dots, J \quad (2)$$

and

$$x_i^l \leq x_i \leq x_i^u \quad (3)$$

For the minimum weight design of structures modeled by bar and membrane plate elements the equations may be given by:<sup>2</sup>

$$f(x)_i = \sum_{i=1}^n A_i x_i \quad (4)$$

$$g_j(x) = \left[ \sum_{i=1}^n E_{ij} / x_i - U_j / U_j \right] \leq 0. \quad (5)$$

Resizing is generally accomplished in Mathematical Programming procedures by letting

$$x_i^{r+1} = x_i^r + \alpha^r S_i^r \quad (6)$$

where  $\alpha^r$  the step size is usually selected arbitrarily and  $S_i^r$  is the direction of movement.

Optimality criteria (OC) methods resize the structure based on a solution of the problem

$$f_{,i} + \sum_{j \in J_A} \lambda_j g_{j,i} = 0 \quad (7)$$

by methods of successive substitutions where

$$x_i^{r+1} = x_i^r S_i^r (\eta \lambda_j) \quad j \in J_A. \quad (8)$$

Here  $\eta$  is "resizing parameter" and  $J_A$  is a set of "active" constraints. For this report the meaning of the  $\eta$  parameter is that of the similar symbol of Ref. 5. Procedures which compute the values of the  $\lambda$  set considering the equations for  $\lambda$  as coupled will be referred to as "generalized" OC methods<sup>3,4</sup> and those where the  $\lambda$  are assumed uncoupled as "simple" OC methods<sup>2</sup>.

## II. OBJECTIVES

There are two distinct projects associated with this research. These are:

1. A preliminary investigation of the effectiveness of a strategy to reduce divergence and oscillation in large scale optimal structural synthesis procedures.

2. The development of a primal Mathematical Programming (MP) procedure suitable for large scale structural synthesis.

### III. OSCILLATION AND DIVERGENCE CONTROL

III.1 Background. Almost all optimization methods suitable for large scale structural synthesis require the selection of a constraint set for inclusion into the resizing problem. This is usually done by including all those constraints where<sup>2,3,5</sup>

$$g_j \geq -e_j. \quad (9)$$

Here  $e_j$  will be referred to as the constraint "band width". The inclusion of too many constraints results in an unnecessary increase in the computational effort required for the solution of the resizing problem. For example, this effort can be substantial and may greatly exceed the reanalysis effort in general OC procedures<sup>6</sup>. Excess constraints may also "overconstrain" the problem producing heavier designs.

On the otherhand selection of band widths that are too small may lead to the major violation of a constraint that was not included in the resizing problem resulting in an increase in weight after scaling<sup>2</sup> or other boundary restoration.

A major difficulty in the use of optimization methods is that no rigorous, or even reasonably reliable, efficient procedure has been formulated for band width selection. The desire to reduce the computational effort and avoid the overconstraint problem usually results in selection of narrow band widths and thus in occasional, or even frequent, problems with oscillation or divergence<sup>5,7</sup>.

Oscillation or divergence resulting from such problems will be referred to here as "primary oscillation". In addition to this mode, oscillation can result from too large a value of the resizing parameter or step size. Such oscillation will be referred to as "secondary oscillation".

III.2 Procedure for dealing with primary oscillation. The basic concept here is to introduce a potentially active constraint set  $J_p$  where  $j \in J_p$  if

$$-e_{1j} \geq g_j \geq -e_{2j} \quad (10)$$

$$e_{2j} > e_{1j} \quad (11)$$

These constraints are not included directly in the resizing problem. However, if after resizing it appears, based on the gradient information associated with these constraints, that any of them may be violated the resizing step is shortened in an attempt to avoid this violation.



Where

$$\Delta x_i^r = x_i^{r+1} - x_i^r \quad (12)$$

the estimated value of  $g_j^{r+1}$  is given by

$$g_j^{r+1} = g_j^r + \sum_i g_{j,i}^r \Delta x_i^r \quad (13)$$

If any

$$g_j^{r+1} > 0 \quad (14)$$

compute a step shortening quantity  $K_j$  such that a move would produce a value of  $g_j^{r+1} = 0$  for these constraints. Thus

$$K_j = -g_j^r / \sum_i g_{j,i}^r x_i^r. \quad (15)$$

Call the smallest of these  $K^*$  and redefine  $x_i^{r+1}$  as

$$x_i^{r+1} = K^* (x_i^r + x_i^r) \quad (16)$$

III.3 Procedure for dealing with secondary oscillation. If a weight increase results after resizing and the sets  $J_A^{r+1}$  and  $J_P^{r+1}$  contain no constraints that were not in either  $J_A^r$  or  $J_P^r$  then it is assumed that the resizing parameter is too large. Design  $r+1$  then discarded and a new resizing move made where

$$\eta^{r+1} = r/2. \quad (17)$$

or

$$\alpha^{r+1} = r/2.$$

III.4 Termination Procedure. The resizing process is terminated if

$$|f(x^{r+1}) - f(x^r)| / f(x^r) \leq C_1 \quad (18)$$

where  $C_1$  is a convergence criteria or if

$$\eta < C_2 \quad (19)$$

where  $C_2$  is the minimum resizing parameter size.

III.5 Added Storage Requirements and Computational Effort. Once a step reduction value  $K_j$  is computed all the gradient information associated with that constraint may be discarded. Thus it is only necessary to store one set of constraint gradient components rather than the gradients of all constraints in the potentially active set. Furthermore, in many procedures once the quantities associated the solution of the resizing problem are computed the gradients of the active set are no longer needed. Some part of the active constraint gradient storage may therefore be used for the particular potentially active constraint being considered. Thus, no additional storage is required for the primary oscillation control procedure in such instances.

In the case of OC methods using the secondary oscillation control procedure if one is to formulate, a new resizing problem at a point  $x_1^r$  using a smaller  $n$  computational effort may be reduced if information associated with the  $\lambda$  set of the former resizing problem is saved. This information is derived from the gradients of the active constraint set. Since active constraint set gradient storage is no longer needed this resizing problem information may be stored in its place. Furthermore, since the active constraint set gradients require much more storage than the resizing information this extra storage may be used for some or all of the potentially active constraint gradients. Thus, with appropriate information storage these procedures should normally not require a substantial increase in storage capacity.

The calculation of the additional potentially active constraint gradients requires additional computational effort at each redesign cycle. This additional effort is however usually small compared to the total computational effort and much smaller than the effort wasted by bad moves resulting from failure to consider, at all, these potentially active constraints.

#### IV. MATHEMATICAL PROGRAMMING (MP) ALGORITHM

IV.1 Background. Mathematical programming procedures are considered to be the most general and easily applied of the various optimization procedures. They avoid some of the difficulties associated with OC procedures<sup>8</sup>. MP procedures are however generally considered to be poorly suited to large structural synthesis problems without use of approximation techniques, at least in their primal form, due to the relatively large number of reanalysis typically required for most procedures<sup>4</sup>.

It has been the feeling of this writer that the view that MP procedures are too inefficient for large scale structural synthesis has some justification in the case of most existing methods. It was also felt, however, that should be relatively easy to construct a procedure based on the method of feasible directions<sup>9</sup> which will, on the basis of total computational effort, be competitive with OC procedures. Such a procedure would provide the flexibility to treat problems poorly suited to OC methods<sup>8</sup>. An attempt at such a procedure is presented here.

IV.2 The Feasible Direction (FD) MP Procedure. The feasible direction problem is usually formulated:<sup>1</sup> Find  $S_i$  and  $\sigma$  so as to

$$\text{Maximize } \sigma \quad (20)$$

where

$$S^T \nabla f(x_i) + \sigma \leq 0 \quad (21)$$

and

$$S^T \nabla g_j(x_i) + W_j \sigma \leq 0 \quad j \in J_A \quad (22)$$

$$-1 \leq S_i \leq 1 \quad (23)$$

where  $S$  consisting of components  $S_i$  is the "feasible direction" of movement,  $\sigma$  is a dummy variable and  $W_j$  a weighting parameter.

Equations (20) and (21) state that on the basis of the linearized functions the solution to this problem will produce a maximum possible improvement in  $f(x_i)$ . Equations (20) and (22) state for  $W_j = 0$  there will be no movement toward constraint violation and where  $W_j > 0$  there will be movement away from violation. Equation (23) is used to eliminate unbounded solutions. This is a linear programming problem and may efficiently be solved by one of many well developed procedures.

Here one has the problem of determining an appropriate active constraint set to include in Eqn. (22). Too large a band width will overconstrain the problem by forcing movement essentially parallel to or away from, a constraint that may not be critical. Too small a band width may produce serious violation of a constraint that was not considered in the direction finding problem of Eqns. (20-23).

The weighting parameter in most feasible directions methods is given as positive in order to avoid constraint violation. Such violation will occur on the convex constraints which are usually encountered in structural design. The central idea of the improved method is to set  $W_j$  such that it will produce movement toward the constraint if one is near but not on a critical constraint. This eliminates the overconstraining effect of too large a band width. Thus replace the  $W_j \sigma$  term in (22) with a term producing movement toward the constraint and thereby rewrite Eq (22) as

$$\alpha^T S^T \nabla g_j^T(x_i) < g_j^T. \quad (24)$$

It may be seen that left hand side will produce an estimated change at most equal to the constraint value. Then after resizing the value of the constraint will move toward zero or even near zero if the constraint

is critical. In other words, if movement toward violation of the constraint improves the design, Eq. (24) will limit that movement only to the extent required to avoid violating the constraint. Thus, the redesign move will be influenced only by constraints expected to be critical.

It should be noted this procedure may also produce oscillation or divergence resulting from deficiencies in band width or  $\eta$  selection the same procedures described earlier for control of oscillation and divergence may of course be used here.

The value for  $\alpha^r$  for this study is given by

$$\alpha^r = \eta^r |S^{r-1}| \nabla f^r \quad (25)$$

where  $|S_1^0| = \sqrt{I}$  and  $\eta^0$  is arbitrarily selected. Such a value of  $\eta$  will tend to produce a change in the objective function of approximately  $100\eta^r$  percent if an S move, similar to  $S^{r-1}$ , is made in the  $\nabla f$  direction. Thus  $\eta = .5$  would tend to produce about a 50% change in weight if one moved in the  $\nabla f$  direction. Since the direction finding problem would produce deflected away from  $\nabla f$  by the constraints, the change after an actual move would be substantially less than that produced by a move in the  $\nabla f$  direction. This would be particularly true during the later stages of the search were the design is highly constrained.

If a redesign move fails to produce a weight reduction  $\eta^r$  is reduced and design  $x^{r+1}$  is discarded as in Section III.3. A new design is then generated using

$$\eta^{r+1} = \eta^r / 2. \quad (26)$$

When this occurs the band widths are also narrowed by setting

$$e_j^{r+1} = e_j^r / 2 \quad (27)$$

The  $e_j^0$  are arbitrarily selected. Similarly these cuts are made if  $S = 0$  is the solution<sup>9</sup>.

The design is restored to the feasible-infeasible boundary by scaling as in Ref. 2 or by some other procedure where the problem is not of the form of eqns (4-5)<sup>9</sup>.

Termination is accomplished by the procedure of Section III.4.

## V. EXAMPLES

V.1 Problems. The two ten bar truss problems posed by Venkayya et al<sup>2</sup> are repeated in this preliminary study using the parameters of Ref. 5. The stress "constrained" problem involves the minimum weight design of the

an indeterminate structure under a single loading condition where the stress in all members must be held at or below one of two specified values. The optimal design in this structure weighs 1,497.60 lbs. In the displacement constrained problem two additional constraints are placed on the deflection of two joints and only one stress limit value is used. Side constraints [eqn (3)] are used for all variables.

The stresses and deflections are determined by a finite element analysis the results of which may be used to develop the constraint gradient information by means of the virtual unit load method<sup>2</sup>.

These are now classic benchwork problems and represent a moderately difficult challenge for any proposed structural optimization scheme. The stress constrained problem has 8 (of 10) active stress constraints in addition to several active side constraints. The displacement constrained problem has two local minima one of which is constrained by the two displacement constraints (this design weighs 5,077.6 lbs) and one by a stress and one displacement constraint (this design weighs 5,061 lbs). Several side constraints are critical in both local optima.

V.2. Procedures. The generalized OC procedures described by Khot et al<sup>5</sup> as the exponential [eq (7) of Ref. 5], and linear [eq (9) of Ref. 5] recursion forms was modified to use the oscillation control techniques described here in. In addition this procedure was also adapted to use the multiple iteration Newton Raphson procedure described by Khot et al in Ref. 7. The procedure from Ref. 7 is remarkably similar in that of Ref. 4 except for the method of solution of the  $\lambda$  problem. The oscillation control methods were added to the experimental program used to obtain the results given in Ref. 5 and 7 to generate the results contained herein.

The MP program used here was also developed from this same experimental program by replacing the resizing portions of the program. It did not employ the primary oscillation control technique.

## VI. RESULTS OF USE OF OSCILLATION AND DIVERGENCE CONTROL PROCEDURE

VI.1 Primary Oscillation Control. Table 1 illustrates the application of this procedure to two cases where serious oscillation was experienced using OC procedures<sup>5,7</sup>. Both involve the stress constrained problem. In the first case the exponential recursion form with multiple iterations solution for the  $\lambda$  set was employed and in the other the exponential form

TABLE 1. Design Sequence, Stress Constrained 10 Bar Truss Problem With and Without Primary Oscillation Control.  $\eta = 0.5$ , Weight in lbs.

<u>Design No.</u>	<u>Linear Form</u> <u>Multiple <math>\lambda</math> Iterations</u>		<u>Exponential Form</u> <u>Single <math>\lambda</math> Solution</u>	
	<u>No. Control</u>	<u>Control</u>	<u>No. Control</u>	<u>Control</u>
1	3,435	3,435	3,435	3,435
2	4,452	2,059	3,118	2,169
3	2,342	1,959	4,870	1,991
4	7,221	1,801	2,809	2,292
5	8,576	1,664	2,746	1,838
6	2,689	1,654	4,854	1,876
7	1,662	1,540	2,840	1,683
8	2,229	1,532	2,169	1,656
9	1,704	1,526	2,219	1,597
10	1,626	1,521	2,976	1,574
11	23,992	1,516	3,871	1,558
12	11,185	1,512	2,969	1,548
13	4,694	1,508	3,370	1,539
14	9,542	1,505	2,796	1,532
15	4,948	1,502	4,340	1,525
16	14,146	1,499	2,496	1,520
17	6,914	1,498	3,880	1,515
18	6,998	1,532	2,701	1,510
19	10,665	1,497.60	4,125	1,507
20	5,667	1,498	2,3122	1,504
21		1,497.60		1,501
22		1,498		1,678
23		1,497.60		1,595
24		1,497.62		1,640
25		1,497.60		1,674

with single solution for the set is used. It may be seen that the use of this procedure did in fact control primary oscillation.

Two other test cases were used in which OC the procedures described in Ref. 5 and 7 were induced to oscillate badly by use of an excessively large resizing parameter ( $\eta = 1$ ). The use of the primary oscillation control again suppressed this oscillation mode.

VI.2 Secondary Oscillation Control. Table 2 illustrates the application of the secondary procedure. With only primary control one sees here secondary oscillation at design number 4-6 and divergence at design number 20. It may be seen that the reduction in the resizing parameter produces convergence to the optimal design.

This procedure was found satisfactory only with the linear recursion forms. It was found unsatisfactory for the exponential form since the assumption that the  $\lambda$  problem formulated on the basis of the linear recursion relation may be used with the exponential form was not valid at small  $\eta$  values. Because of this all subsequent numerical experiments are based on the linear recursion forms.

All runs used  $e_{2j} = 2 e_{1j}$  to define the potentially active constraints. The quantities  $e_{1j}$  for the active constraints are defined by the procedure of Ref. 5. Termination constraints used for all runs were  $C_1 = 10^{-6}$ ,  $C_2 = 0.01$ .

VI.3 Use of a Large Initial Resizing Parameter. Since the early experiments indicated that these controls would inhibit oscillation that would usually occur with excessive value of  $\eta$  an experiment was performed to investigate the possibility of using a large initial value of this parameter so as to speed convergence. The result is shown in Tables 3 and 4. These results fail to support the hypothesis that a large initial  $\eta$  is advantageous.

VI.4 Convergence of Multiple and Single Problem Solutions Procedures. It may also be seen from Tables 3 and 4 that the multiple  $\lambda$  iteration procedure possesses much better convergence properties than the single  $\lambda$  solution procedure. In fact the single  $\lambda$  solution procedure fails on the displacement constrained problem. It also fails on this problem when no oscillation controls are used.

An oscillation control procedure is important in allowing exploitation of the multiple  $\lambda$  iteration approach because of the oscillation problems with this method encountered in earlier studies. It is interesting

TABLE 2. Design Sequence, Stress Constrained 10 Bar Truss Problem Single Solution, Linear Form  $\eta = 0.5$  at Start, Weight in lbs.

<u>Design No.</u>	<u>Primary Control Only</u>	<u>Both Controls</u>
1	3,435	3,435
2	2,444	2,444
3	1,836	1,836
4	2,294	2,294*
5	1,863	1,726
6	2,061	1,653
7	1,727	1,586
8	1,718	1,556
9	1,569	1,539
10	1,541	1,529
11	1,524	1,523
12	1,518	1,518
13	1,513	1,513
14	1,509	1,509
15	1,506	1,506
16	1,503	1,503
17	1,501	1,501
18	1,499	1,499
19	1,497.6	1,497.74
20	1,656	1,656*
21	1,685	1,572*
22	1,681	1,534*
23	1,677	1,516
24	1,677	1,497.67
25	1,678	1,506*
26		1,497.63
27		1,502*
28		1,497.61*

\*Halved resizing parameter at this redesign cycle.



TABLE 3. Comparison of Design Sequences Using Different Initial Resizing Parameters, Stress Constraints 10 Bar Truss, Weight in lbs.

Design No.	Multiple $\lambda$ Iterations		Single $\lambda$ Solution	
	$\eta=1$ at Start	$\eta=0.5$ at Start	$\eta=1$ at Start	$\eta=0.5$ at Start
1	3,435	3,435	3,435	3,435
2	2,128	2,059	2,740	2,444
3	1,915	1,959	1,825	1,836
4	1,772	1,801	1,719	2.294*
5	1,638	1,664	1,628	1,726
6	1,593	1,654	1,580	1,653
7	1,532	1,540	1,548	1,586
8	1,527	1,532	1,531	1,555
9	1,521	1,526	1,519	1,539
10	1,512	1,521	1,509	1,529
11	1,508	1,516	1,499	1,523
12	1,505	1,512	1,517*	1,518
13	1,502	1,508	1,638*	1,513
14	1,500	1,505	1,497.9	1,509
15	1,558*	1,502	1,572*	1,506
16	1,497.62	1,499	1,502	1,503
17	1,497.60	1,498	1,497.7	1,501
18	1,497.60†	1,532*	1,516*	1,499
19		1,497.63	1,506**	1,498
20		1,497.60		1,656*
21		1,497.60†		1,572*
22				1,534*
23				1,516*
24				1,497.67
25				1,506*
				(28)1,497.61†

\*Halved resizing parameter.

†Terminated by convergence specification.

\*\*Terminated by minimum resizing parameter specification.

TABLE 4. Comparison of Design Sequences Using Different Initial Resizing Parameters, Displacement Constrained 10 Bar Truss, Weight in lbs.

Design No.	Multiple $\lambda$ Iterations		Single $\lambda$ Solution	
	$\eta=1$ at Start	$\eta=0.5$ at Start	$\eta=1$ at Start	$\eta=0.5$ at Start
1	8,266	8,266	8,266	8,266
2	6,893	6,646	7,146	6,667
3	5,816	5,824	6,932	6,196
4	5,593	5,703	6,641	5,947
5	5,466	5,597	5,376	5,759
6	5,327	5,471	5,221	5,628
7	5,358	5,353	5,216	5,516
8	5,505*	5,195	5,162	5,401
9	5,200	5,079.6	5,179	5,280
10	5,094	5,078.9	5,129	5,187
11	5,076	5,084.4*	5,118	5,143
12	5,066	5,077.0	5,115	5,115
13	5,061.0	5,076.7	5,113	5,108
14	5,060.87	5,076.7	5,112	5,090.4
15	5,060.85		5,117	5,104.4*
16	5,060.85		5,111.5	5,094.2
17			5,111.35	5,089.4
18			5,111.31	5,088.0
19			5,111.253	5,086.9
20			5,111.247	5,086.0
25			5,098.7*	5,083.6*
30			5,094.6	5,082.9†
35			5,093.4	
50			5,093.3	

\*Halved resizing parameter.

†Terminated by convergence specification.

to note that although Ref. 7 mentions the potential of this procedure it is not used in the later study of OC methods<sup>5</sup>.

VI.5 Comparison of Ordinary and Inverse Variables. The results of this study are shown in Table 5. There is no apparent advantage associated with the use of inverse variables.

TABLE 5. Comparison of Inverse and Ordinary Variables Using Multiple  
Iteration Linear Form Procedure,  $n = 0.5$  at Start, Weight in lbs.  
10 Bar Truss

<u>Design No.</u>	<u>Inverse</u>	<u>Ordinary</u>	<u>Inverse</u>	<u>Ordinary</u>
1	3,435	3,435	8,266	8,266
2	2,098	2,059	6,912	6,645
3	1,903	1,959	6,438	5,824
4	1,662	1,801	6,194	5,703
5	1,624	1,664	6,019	5,597
6	1,537	1,654	5,877	5,471
7	1,518	1,540	5,750	5,353
8	1,513	1,532	5,628	5,195
9	1,509	1,526	5,511	5,080
10	1,503	1,521	5,402	5,078.9
11	1,501	1,516	5,307	5,084*
12	1,593*	1,512	5,182	5,077.0
13	1,497.67	1,508	5,101	5,076.7
14	1,497.60	1,505	5,094	5,076.7
15	1,565*	1,502	5,081	
16	1,497.60	1,498.8	5,094	
17		1,497.9	5,081	
18		1,532*	5,0770	
19		1,497.63	5,076.7	
20		1,497.60	5,107*	
21		1,497.60	5,076.7	

\*Halved resizing parameter.

## VII. PERFORMANCE OF THE FEASIBLE DIRECTIONS ALGORITHM

The results of this algorithm on the example problems using various values of  $\eta^0$  are shown in Table 6. In all cases  $e_j^0 = 0.5$ . A small  $\eta^0$  produces small changes in the initial designs. A large  $\eta^0$  on the other-hand produces early oscillation associated with the need for step size reduction. This of course is similar to the situation in OC procedures.

Further development can undoubtedly substantially improve performance. For example the algorithm described in section V was modified such that; 1) the equations (21) and (28) are invoked (step size is reduced) after the change in weight on scaling is greater than the net decrease in weight after redesign and scaling; 2) the  $S^{r-1}$  in eq. (25) replaced by  $S^*$  where  $S^*$  is obtained after iterative solution of the feasible direction problem [eqns. (19-25)] until  $|S^*| = |S^r|$ . This modification produced a dramatic improvement in performance (see Table 6) on the sole problem on which it was tested.

## VIII. COMPARISON OF THE OC AND FD PROCEDURES

On the basis of this limited study if one uses similar values of the resizing parameter it appears that the MP procedure requires fewer reanalyses for convergence than the single  $\lambda$  solution OC procedure even after the oscillation control improvements are made to the latter. The multiple  $\lambda$  iteration approach seems to possess superior convergence properties when these controls are used. It appears, however, that it may be relatively simple to greatly improve the performance of the MP procedure to the point where it is comparable to the multiple  $\lambda$  iteration OC procedure.

On the basis of computational effort required for convergence the picture is somewhat different. Analysis and resizing times are shown in Table 7. The analysis time is the CPU time required to do the finite element analysis. The resizing time includes the time required to compute the necessary derivatives and set up and solve either the  $\lambda$  or the feasible direction problem.

It may be seen that the resizing time required for the MP procedure is similar to the single  $\lambda$  solution OC method which is much less than the multiple  $\lambda$  iteration approach. Furthermore, it is the resizing effort that dominates. On larger problems one would expect a similar situation.

TABLE 6. Design Sequences for Feasible Directions Algorithm Using Different Resizing Parameters, Weight in lbs. 10 Bar Truss.

Design No.	Stress Constrained			Displacement Constrained			
	0.1	0.2	0.5	0.1	0.2	0.5	0.5
1	3,435	3,435	3,435	8,266	8,266	8,266	8,266
2	3,166	2,885	2,224	7,770	7,394	7,006	7,006
3	2,884	2,455	2,722	7,040	6,837	7,642	6,610
4	2,641	2,096	1,818	6,801	6,629	7,608	6,385
5	2,448	1,817	2,300	6,615	6,402	6,331	6,171
6	2,279	2,006	1,787	6,466	6,097	6,217	5,976
7	2,125	1,703	1,677	6,309	5,826	5,796	5,854
8	1,854	1,693	1,604	6,150	5,668	5,492	5,564
9	1,736	1,623	1,557	6,002	6,358	5,225	5,398
10	1,792	1,571	1,528	5,827	5,512	5,164	5,176
11	1,671	1,535	1,507.6	5,637	5,172	5,281	5,115
12	1,644	1,516	1,501.4	5,396	5,114	6,225	5,739
13	1,624	1,742	1,737	5,178	5,090	5,175	5,066.9
14	1,606	1,506	1,498.3	5,112	5,088	5,125	5,126
15	1,585	1,737	1,498.4	5,169	5,090	5,108	5,091
16	1,567	1,501	1,497.9	5,082	5,081	5,099	5,067.6
17	1,549	1,725	1,498.5	5,079	5,084	5,095	
18	1,524	1,498	1,497.7	5,077	5,079.6	5,088	
19	1,515	1,725	1,498.4	5,078	5,079.6	5,085	
20	1,505	1,549	1,497.6	5,077.0	5,079.4	5,081	
25	1,506	1,497.6		5,077.6	5,078.6	5,077.1	
30	1,497.6			5,076.7	5,077.9	5,076.7	
35					5,077.6		
50					5,076.7		

†Modified Algorithm.

TABLE 7. Typical Analysis and Resizing Times, CPU Sec CYBER 74

	<u>Multiple <math>\lambda</math> Iteration</u>		<u>Single <math>\lambda</math> Solution</u>		<u>Feasible Direction</u>	
	<u>Analysis</u>	<u>Resizing</u>	<u>Analysis</u>	<u>Resizing</u>	<u>Analysis</u>	<u>Resizing</u>
Ten Bar Truss Stress Constrained	.022	1.996	.022	.1330	.073	.274
Ten Bar Truss Displacement Constrained	.021	.379	.022	.071	.022	.112
Two Hundred Bar Truss	2.351	17.44	2.341	.708	-	-

Furthermore, on large problems a more efficient but more complex linear programming procedure which ignores zero matrix entries rather than the simple procedure used here should substantially reduce the resizing effort making the MP procedure more attractive.

#### IX. CONCLUSION

Much more work needs to be done to verify the preliminary results developed here after further refinement of the concepts presented. The results of this work, however, supports the initial assumptions that oscillation problems associated with many optimization methods may easily be greatly reduced and that a simple primal MP procedure without approximations can be competitive with OC procedures for finite element based structural synthesis.

On the basis of this preliminary study the MP procedure seems more attractive than the OC procedures on large problems with many active constraints due to the large effort required to set up the  $\lambda$  problem<sup>6</sup>. On problems with very few active constraints the ability of the OC procedures to produce very large initial weight reductions makes these procedures attractive. Additional work needs to be done however to reduce the resizing computation effort of the multiple  $\lambda$  iteration approach to allow exploitation of its superior convergence properties.

These conclusions are of course only tentative and are based on very limited evidence. The importance of these initial successes however justifies expanded research on these techniques.

#### X. RECOMMENDATIONS

The preliminary results of this early research are quite encouraging and justify further study since these methods, if successful, represent major advances. The following additional research is therefore recommended:

1. Application of the oscillation control techniques to a simple OC procedure.
2. Refinement of the techniques. For example the development or improvement in the methods of band width and step size determination or specification.
3. Treatment of additional static problem examples.
4. Treatment of dynamic problem examples.
5. Treatment of example problems with local buckling constraints.



6. Comparison of the procedures developed as the result of this research with important large scale synthesis capabilities such as the ACCESS3 and OPTSTAT<sup>10</sup> codes.
7. If justified, the incorporation of successful new methods into a formal structural synthesis program for general distribution.

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